Resonant charging equations

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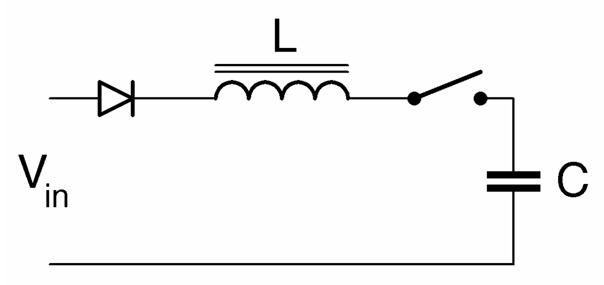
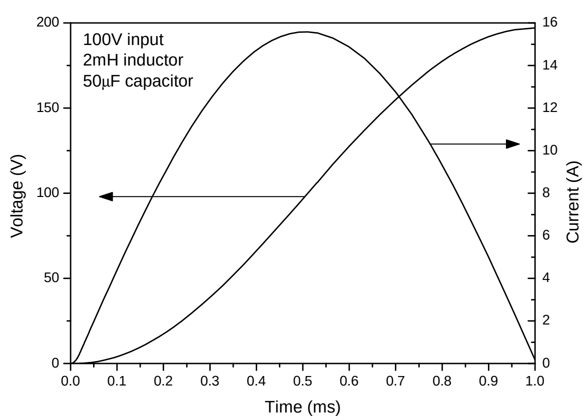
Introduction

Resonant charging is a neat way of rapidly charging capacitors in pulse applications. It is highly efficient compared with resistive charging (large power loss in the resistor). Here's a simplified circuit:



When the switch is closed, the full input voltage is applied across the inductor since the capacitor is initially discharged. Current starts to build up in a sinusoidal fashion and the voltage begins to rise on the capacitor. Once the current has completed a full half cycle, the capacitor is fully charged at twice the input voltage and the series diode prevents discharge back into the power supply. Here's the voltage and current waveforms for some sample component values:



Charge time

The capacitor is fully charged in one half cycle of oscillation. The standard equation for the resonant frequency of an LC circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

so the oscillation period is

$$T = 2\pi\sqrt{LC}$$

Therfore, the charge time half this or

$$\boxed{t_{charge} = \pi\sqrt{LC}}$$

Plugging in values of $L = 2\text{mH}$ and $C = 50\text{uF}$ gives $t_{charge} = 1\text{ms}$, in agreement with the observed value from the graph.

Peak inductor current

The inductor current follows a half-sinusoidal waveform. The duration of the half sinusoid is t_{charge} so we can express the current as a function of time as

$$I = I_{peak}\ Sin\left(\frac{1}{\sqrt{LC}}t\right)$$

where I_{peak} is the peak inductor current. How did the factor of $1/\sqrt{LC}$ come about? Well, when $t = t_{charge} = \pi\sqrt{LC}$, the inductor current is $I = I_{peak}\ Sin\left(\pi\right) = 0$, which is correct. I just chose the $1/\sqrt{LC}$ so that this worked out.

The power drawn from the supply voltage is given by $V * I$, which is:

$$P = V_{in}I_{peak}\ Sin\left(\frac{t}{\sqrt{LC}}\right)$$

If we integrate this from 0 to t_{charge}, we will get the total energy drawn from the supply voltage:

$$\begin{aligned}
E &= V_{in}I_{peak}\int_0^{\pi\sqrt{LC}} Sin\left(\frac{t}{\sqrt{LC}}\right)dt \\
&= V_{in}I_{peak}\left(-\sqrt{LC}\right)\left[Cos\left(\frac{t}{\sqrt{LC}}\right)\right]_0^{\pi\sqrt{LC}} \\
&= -V_{in}I_{peak}\sqrt{LC}\left[-1-1\right] \\
&= 2V_{in}I_{peak}\sqrt{LC}
\end{aligned}$$

But wait a minute - we know that the capacitor is charged to twice the input voltage, so the energy stored in it must be $\frac{1}{2}C\left(2V_{in}\right)^2 = 2CV_{in}^2$. These two energies must obviously be equal, and by equating them we can obtain an expression for the peak inductor current:

$$\begin{aligned}
2V_{in}I_{peak}\sqrt{LC} &= 2CV_{in}^2 \\
I_{peak}\sqrt{LC} &= CV_{in} \\
I_{peak} &= \frac{CV_{in}}{\sqrt{LC}}
\end{aligned}$$

$$\boxed{I_{peak} = V_{in}\sqrt{\frac{C}{L}}}$$

Plugging in our values again of $V_{in} = 100\text{V}$, $C = 50\text{uF}$ and $L = 2\text{mH}$, we get $I_{peak} = 15.8\text{A}$, which again agrees with the graphs.

Peak inductor energy

To find the maximum energy stored in the inductor, which is very important when choosing a suitable core and gap, we simply use the usual equation $\frac{1}{2}LI^2$ with the peak inductor current:

$$E_{ind.peak} = \frac{1}{2}LV_{in}^2\frac{C}{L}$$

$$\boxed{E_{ind.peak} = \frac{1}{2}CV_{in}^2}$$

Interestingly, this does NOT depend on the actual inductance and is one quarter of the output capacitor energy (which is $2CV_{in}^2$).

Summary

Here's the equations again:

$$\begin{aligned} t_{charge} &= \pi\sqrt{LC} \\ I_{peak} &= V_{in}\sqrt{\frac{C}{L}} \\ E_{ind.peak} &= \frac{1}{2}CV_{in}^2 \end{aligned}$$